Six variations on fair wages and the long-run Phillips curve

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Abstract

The present paper explores the connection between inflation and unemployment in different models with fair wages both in the short and in the long runs. Under customary assumptions regarding the sign of the parameters of the effort function, more inflation lowers the unemployment rate, though to a declining extent. This is because firms respond to inflation - that spurs effort by decreasing the reference wage - by increasing employment, so to maintain the effort level constant, as implied by the Solow condition. Under wage staggering this effect is stronger because wage dispersion magnifies the impact of inflation on effort. A stronger effect of inflation on unemployment is also produced under varying as opposed to fixed capital, given that in the former case the boom produced by a monetary expansion is reinforced by an increase in investment. Our baseline results are robust to the adoption of a model based on reciprocity in labour relations. Therefore, we provide a new theoretical foundation for recent empirical contributions finding negative long- and short-run effects of inflation on unemployment.

Keywords: efficiency wages, money growth, long-run Phillips curve, trend inflation, wage staggering, reciprocity in labour relations.

JEL classification codes: E3, E20, E40, E50.
1 Introduction

The economic literature has recently witnessed a flourishing of contributions nesting an efficiency wages framework into business cycle models. Earlier models were proposed within the real business cycle (RBC) realm. Danthine and Donaldson (1990), for instance, showed that efficiency wages within a RBC model can produce structural unemployment, but not wage stickiness over the economic cycle. With difference to Danthine and Donaldson (1990), which focused on a gift exchange model, Uhlig and Xu (1995) and Gomme (1999) adopted a shirking model. However, in a rather similar way, they found that wages tend to be too volatile and employment not enough so over the cycle. In Kiley (1997) efficiency wages generate completely a-cyclical real wages, but not a greater endogenous price stickiness, because the a-cyclical real-wage requires countercyclical effort and hence a procyclical marginal cost.

Collard and de la Croix (2000) showed that, once including past compensations into the reference wage, an efficiency wages/RBC model can replicate wage acyclicalty. Along similar lines, Danthine and Kurmann (2004) proposed a model combining efficiency wages of the gift exchange variety - also termed fair wages - with sticky prices, showing that it can well account for the low correlation between wages and employment, also displaying a greater internal propagation of monetary shocks than standard New Keynesian models. Danthine and Kurmann (2008), inspired by Rabin (1993), explicitly modelled the psychological benefits arising from gift exchanges between firms and workers in terms of remuneration and effort respectively. Danthine and Kurmann (2010) incorporated a reciprocity-based model of wage determination into a dynamic general equilibrium model, which was then estimated on U.S. data. They highlighted that wage setting is driven more by rent-sharing and past wages, than by aggregate employment conditions.

Alexopoulos (2004, 2006, 2007) developed a model in which shirkers are not dismissed once detected. They, instead, forgo an increase in compensa-
tion. Under these assumptions it was showed that an efficiency wage model can well replicate empirical evidence regarding the response of the economic system to technological, fiscal and monetary shocks.

The present paper, instead, focuses on the long-run and short-run implications of efficiency wages for the connection between unemployment and inflation under trend money growth within a dynamic general equilibrium framework. In so doing, we extend a literature that so far investigated the long-run and, to a lesser extent, the short-run effects of money growth by resorting only to models with wage/price stickiness. Pioneering contributions on this issue were King and Wolman (1996) and Ascari (1998). The former study considered a model with a shopping time technology and it obtained a number of different results, among which there is that long-run inflation reduces firms’ markup, boosting the level of output. Ascari (1998), instead, showed that in wage-staggering models money can have considerable negative non-superneutralities once not considering restrictively simple utility and production functions. Deveraux and Yetman (2002) focused on a menu cost model. An analysis of dynamic general equilibrium models under different contract schemes in presence of trend inflation was offered in Ascari (2004). Graham and Snower (2004), instead, examined the microeconomic mechanisms underlying this class of models. In presence of Taylor wage staggering, in a monopolistically competitive labour market, they highlighted three channels through which inflation affects output: employment cycling, labour supply smoothing and time discounting. The first one consists in firms continuously shifting labour demand from one cohort to the other according to their real wage. Given that different labour kinds are imperfect substitutes, this generates inefficiencies and it tends to create a negative inflation-output nexus. The second one is that households demand a higher wage in presence of employment cycling given that they would prefer a smoother working time. This decreases labor supply and aggregate output. Finally under time discounting the contract wage depends more on the current (lower) level of
prices than on the future (higher) level of prices and, therefore - over the contract period - the real wage will be lower the greater is the inflation rate, spurring labour demand and aggregate output. The time discounting effect dominates at lower inflation rates, while the other two effects at higher inflation rates, producing a hump-shaped long-run Phillips curve. The ultimate goal of Graham and Snower (2004) is questioning the customary assumption to identify aggregate demand and supply shocks, namely that the former ones would be temporary and the latter ones not so. As a consequence also the concept of the NAIRU would be unsuitable for a fruitful investigation of the dynamics of the unemployment rate.

Graham and Snower (2004) was extended in a number of different directions. Graham and Snower (2008) showed that under hyperbolic time discounting positive money non-superneutralities are more sizeable than under exponential discounting. Vaona and Snower (2007, 2008) showed how the shape of the long-run Phillips curve depends on the shape of the production function. Finally, Vaona (2010) extended the model by Graham and Snower (2004) from the inflation-output domain to the inflation-real growth one.

We here propose six variations on the theme of efficiency wages and the Phillips curve. In the first one, efficiency wages of the gift exchange variety are coupled with trend money growth, once specifying the reference wage as a function of the unemployment rate, the current individual real wage, the current aggregate real wage and of the current real value of the past aggregate wage. After Becker (1996), this specification has been termed in the literature as social norm case. Being here the reference wage a function of the current real value of the past aggregate wage and not, as in Danthine and Kurmann (2004), of the past real wage, we can highlight the macroeconomic consequences of a peculiar gift exchange between firms and workers that was not investigated so far, though being empirically relevant. Bewley (1999) stresses many times that firms, though not liking wage indexation, are not insensitive to the damages produced by inflation to the purchasing power of
wages. If workers perform well, pay managers will consider fair to offset the negative effect inflation can have on workers’ standard of living\(^1\). This can be conceptualized as a gift exchange: workers elicit effort and firms maintain the purchasing power of their wages. We show that this mechanism can produce sizeable money non-supernaturalities both in the short and long run.

In our second variation, the reference wage is not a function of the current real value of the past aggregate wage, rather of that of the past individual one, as in the personal norm case. Our third model combines Taylor wage stickiness with fair wages of the social norm variety. In this setting, positive money non-supernaturalities turn out to be stronger than under flexible wages\(^2\). The fourth variation extends the first one by considering varying instead of fixed capital. In the fifth and sixth variations, we show that our baseline results also hold in a framework à la Danthine and Kurmann (2008, 2010).

With difference to Graham and Snower (2004, 2008) we provide not only a long-run analysis but also a short-run one, because we think that, even if

\(^1\)To the reader convenience we report some quotations from Bewley (1999). "Other important influences were raises at other firms competing in the same labor markets and changes in the cost of living. Employers wished to protect employees’ standard of living, both to maintain morale and out of a sense of moral responsibility. Many firms did not, however, fully offset increases in living costs in all circumstances" (pp. 160-161). "When hiring someone, I pay them a salary equal to the value of their job. Inflation effectively reduces it, and fairness requires that I offset the reduction. I think that is the way it ought to be. If I hire people at a certain rate, I want to keep that level constant in terms of standard of living" (p. 164). "In deciding on the level of raises, we look at the rate of inflation in the cost of living. It is an indicator of what the competition is doing (...)" (p. 165). "Cost-of-living inflation was a major factor in the determination of raises. [...] The pay of low-performing workers was often allowed to fall behind inflation" (p. 208). "Question: Would a pay cut of 10 percent with no inflation have more impact on employees than a pay freeze with 10 percent inflation? Answer: Both are wage cuts. [...] The company would have to be in trouble. In both cases, people might leave [...]" (p. 209).

\(^2\)Fan (2007) proposed to merge sticky and efficiency wages, but not in an intertemporal optimization framework as we do here.
one cannot identify demand and supply shocks on the basis of their tran-
sience, it will be interesting to investigate how the economic system reacts to
temporary monetary shocks. In other words, transition dynamics does not
lose interest.

Our results can offer a new theoretical foundation for the empirical find-
ings obtained in various recent contributions, that have already been dis-
cussed in Karanassou et al. (2010). A brief review is offered here focusing on
the analyzed countries and time periods, on the adopted econometric meth-
ods and on a common result of theirs, which is particularly relevant to our
analysis.

Karanassou et al. (2003, 2005) bring dynamic multi-equation models to
both European and US annual data from 1977 to 1998 and from 1966 to 2000
respectively. In the former case they rely on panel data methods, while in
the latter one on the three-stage least squares (3SLS) estimator. Karanassou
et al. (2008a) expands the model by Karanassou et al. (2005) by endo-
genizing productivity and financial wealth and deriving the unemployment
rate from labour supply and demand equations. Then they apply a six-
equation structural model to US data running from 1965 to 2000 by using
an autoregressive distributed lags (ARDL) estimator. Model simulation are
finally offered over the period from 1993 to 2000 reaching the conclusion that
money growth put upward pressure on inflation and substantially lowered
unemployment. Rising productivity growth, budget deficit reductions, and
a widening trade deficit played a minor role in inflation and unemployment
dynamics. Karanassou et al. (2008b) bring a structural model to Spanish
annual data from 1966 to 1998 by using both ARDL and 3SLS estimators.
A common result of theirs is that inflation and unemployment are connected
not only in the short-run but in the long-run too. The long-run elasticity
of inflation with respect to unemployment was estimated to be about −3.5,
which was explained by resorting to frictional growth, namely the interplay
between frictions (lagged adjustments) and growth in economic variables. In
the light of our models, this result can be also interpreted as the outcome of efficiency wages mechanisms as explained below.

The rest of this paper is structured as follows. The next section introduces the households’ problem and the government budget constraint, which are common to most of the models here presented. Afterwards, we will introduce the firms’ problem for the social norm case with flexible wages, the personal norm case under flexible wages, the social norm case with wage staggering and the social norm case with varying capital. The seventh section shows that our results hold also adopting a model based on reciprocity on labour relations à la Danthine and Kurmann (2008, 2010). The last section concludes.

In all the cases, we show what is the impact of money growth on both the unemployment and the inflation rates both in the short- and in the long-runs and we discuss the plausibility of our models in order to detect our preferred ones. Introducing capital accumulation at a later stage is not an unusual procedure in the New-Keynesian literature (see for instance Huang and Liu, 2002; Ascari, 2004; Danthine and Kurmann, 2010). Some contributions do not even consider capital accumulation (Ascari 1998; Graham and Snower, 2004, 2008; Danthine and Kurmann, 2008; Ascari and Ropele, 2009). This can be explained by at least two reasons. In the first place, as reminded by Ascari (2004), McCallum and Nelson (1999) argued that it is difficult to specify a capital demand function which is "both analytically tractable and empirically successful". In the second place - similarly to sticky wages/prices models (Ascari, 2004, Vaona, 2010) - the core of our model is in the labour market and capital accumulation turns out to be just a superstructure, not inducing any qualitative change in our results. Therefore, we believe our exposition strategy is the most suited to convey the underlying intuition of our model.
2 The households’ problem and the government budget constraint

We follow Danthine and Kurmann (2004, 2008, 2010), by supposing the economy to be populated by a continuum of households normalized to 1, each composed by a continuum of individuals also normalized to 1. We adopt a money-in-the-utility-function approach to preserve comparability with the trend inflation literature (Ascari 2004, Graham and Snower, 2004, 2008). Households maximize their discounted utility

\[
\max_{\{c_{t+i}(h), B_{t+i}(h), M_{t+i}(h), e_{t+i}(h)\}} \sum_{i=0}^{\infty} \beta^{t+i} E \left( U \left\{ \begin{array}{c}
\quad c_{t+i}(h), n_{t+i}(h) G[e_{t+i}(h)], \\
\quad V \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]
\end{array} \right\} \right)
\]

subject to a series of income constraints

\[
c_{t+i}(h) = \frac{W_{t+i}(h)}{P_{t+i}} n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + M_{t+i-1}(h) \frac{B_{t+i}(h)}{P_{t+i}} + \frac{B_{t+i-1}(h)}{P_{t+i}} \ell_{t+i} + q_{t+i}(h)
\]

where \( \beta \) is the discount factor, \( E \) is the expectation operator, \( U \) is the utility function, \( c_{t+i}(h) \) is consumption of household \( h \) at time \( t+i \), \( B_{t+i}(h) \) are the household’s bond holdings, \( \ell_{t+i} \) is the nominal interest rate, \( n_{t+i}(h) \) is the fraction of employed individuals within the household, \( G[e_{t+i}(h)] \) is the disutility of effort - \( e_{t+i}(h) \) - of the typical working family member, \( V \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right] \) is the utility arising from nominal money balances - \( M_{t+i}(h) \) - over the price level - \( P_{t+i} \). \( W_{t+i}(h) \) and \( T_{t+i}(h) \) are the household’s nominal wage income and government transfers respectively. Finally, \( q_{t+i}(h) \) are profits that households receive from firms.

\(^3\)Feenstra (1986) showed the functional equivalence of money-in-the-utility-function models and liquidity-costs ones.
In this framework, households, and not individuals, make all the decisions regarding consumption, bond holdings, real money balances and effort\(^4\). Individuals are identical ex-ante, but not ex-post, given that some of them are employed - being randomly and costlessly matched with firms independently from time - and some others are unemployed. The fraction of the unemployed is the same across all the families, and so their ex-post homogeneity is preserved.

Note that in our model no utility arises from leisure, therefore individual agents inelastically supply one unit of time for either work or unemployment related activities. Furthermore, after Akerlof (1982), workers, though disliking effort, will be ready to exert it as a gift to the firm if they receive some other gift in exchange, such as a real compensation above some reference level.

Similarly to Danthine and Kurmann (2004), on the basis of the empirical evidence produced by Bewley (1998), we specify the effort function, \(G[e_{t+i}(h)]\), as follows

\[
G[e_{t+i}(h)] = \left\{ e_{t+i}(h) - \left[ \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i+1}(h)}{P_{t+i}} \right] \right\}^2
\] (3)

in the personal norm case and as follows

\[
G[e_{t+i}(h)] = \left\{ e_{t+i}(h) - \left[ \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i+1}(h)}{P_{t+i}} + \phi_4 \log \frac{W_{t+i+1}(h)}{P_{t+i}} \right] \right\}^2
\] (4)

\(^4\)This modelling device is not only common to efficiency wages models (Danthine and Kurmann, 2004, 2008, 2010), it is also used in neo-Keynesian models with search frictions in the labour market (Blanchard and Galí, 2010 on the footsteps of Merz, 1995). Its underlying assumption is full risk sharing and its ultimate goal is to preserve a representative agent setup. Alexopoulos (2004) justifies a similar framework assuming that households can observe individuals’ behavior and that they can punish workers declining job offers by withdrawing income insurance. It would also be possible to think that workers and not households decide how much effort to elicit. However, since all workers within a household are symmetrical, it would not change our results.
in the social norm case\(^5\). \(W_{t+i}\) is the aggregate nominal wage and \(u_{t+i}(h) = 1 - n_{t+i}(h)\) is the unemployment rate. Note that, with difference to Danthine and Kurmann (2004), the nominal (either individual or aggregate) wage at time \(t + i\) is assessed at the prices of time \(t + i\). This assumption does not entail any money illusion. On the contrary, its underlying intuition is that households are aware of the damages that inflation can produce to their living standards and so they are ready to exchange more effort for a pay policy that allows nominal wages to keep up with inflation. More briefly, a higher inflation rate reduces the reference wage.

Throughout the paper, similarly to Danthine and Kurmann (2004), we assume \(\phi_1, \phi_2 > 0\) and \(\phi_3, \phi_4 < 0\). In words a higher household’s real wage and a higher unemployment rate induce more effort. On the other hand, a higher reference wage - be it due to either a higher aggregate wage or a higher real value of past compensation - depresses effort.

Note that, under the hypothesis of an additively separable utility function, utility maximization implies that

\[
G'[e_{t+i}(h)] = 0
\]  

and, therefore, that in the personal norm case

\[
e_{t+i}(h) = \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}}
\]  

\(^5\)An alternative approach to the effort function is the one pursued by Campbell (2006, 2008a and 2008b), which entails a more general functional specification to be linearized at a later stage. However, calibration is less straightforward in this context and economic theorizing is usually followed by a number of numerical exercises where parameters and results display a somewhat large variation. For this reason we prefer to follow Danthine and Kurmann (2004).
and in the social norm case
\[
e_{t+i}(h) = \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}}
\]

Similarly to Danthine and Kurmann (2004), we assume that \( c_{t+i}(h) \) and \( M_{t+i}(h) / P_{t+i} \) enter (1) in logs
\[
U(\cdot) = \log c_{t+i}(h) - n_{t+i}(h) G[e_{t+i}(h)] + b \log \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]
\]

Utility maximization implies
\[
\frac{1}{c_{t+i}(h)} = E \left[ \frac{\pi_{t+i} - c_{t_{i+1}}(h)}{\pi_{t+i+1} c_{t_{i+1}}(h)^{\beta}} \right]
\]
\[
\left( \frac{\mu_{t+i}}{\pi_{t+i}} \right)^{-1} = \frac{c_{t+i}(h)}{c_{t+i}(h)} \left( 1 - \frac{1}{\mu_{t+i}} \right) / \left( 1 - \frac{1}{\mu_{t+i-1}} \right)
\]
where \( \mu_{t+i} \) is the money growth rate and \( \pi_{t+i} \) is the inflation rate. The government rebates its seigniorage proceeds to households by means of lump-sum transfers, \( T_t(h) \):
\[
\int_0^1 \frac{T_{t+i}(h)}{P_{t+i}} dh = \int_0^1 \frac{M_{t+i}(h)}{P_{t+i}} dh - \int_0^1 \frac{M_{t+i-1}(h)}{P_{t+i}} dh
\]

3 First variation: the social norm case

3.1 The long-run

Firms in the perfectly competitive product market hire individuals belonging to all the households to produce their output. Firms maximize their profits - \( P_{t+i} y_{t+i} - \int_{h=0}^1 W_{t+i}(h)n_{t+i}(h)dh \), where \( y_{t+i} \) is output - subject to their
production function - \( y_{t+i} = \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n - 1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n - 1}{\theta_n}} dh \right]^{\frac{\theta_n - 1}{\theta_n}} \), where \( \theta_n \) is the elasticity of substitution among different labour kinds - and to (7), by choosing \( n_{t+i}(h) \) and \( W_{t+i}(h) \). Note that the production function displays decreasing marginal returns to each labour type and constant returns to scale.

The first order condition with respect to \( n_{t+i}(h) \) equates the marginal cost of labour to its marginal product. All households are symmetrical, so we can drop the \( h \) index and write\(^6\)

\[
\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} \tag{12}
\]

whereas the first order condition with respect to \( W_{t+i}(h) \), instead, equates the marginal cost of rising the real wage to the benefit that this induces by increasing effort

\[
\frac{W_{t+i} n_{t+i}}{P_{t+i} y_{t+i}} = \frac{\phi_1}{e_{t+i}} \tag{13}
\]

By substituting (12) into (13), one obtains the well known Solow condition

\[ e_{t+i} = \phi_1 \tag{14} \]

Therefore, firms, maximizing their profits, demand the same effort to all households, across time and independently from the rate of inflation. Furthermore, (14) and the production function, under the condition of households’ symmetry, imply

\[
\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} = \phi_1 \tag{15}
\]

Substitute (14) and (15) into (7) and consider that trend inflation is equal to steady state money growth, \( \mu \), to obtain

\[
\log u = \frac{\phi_0 - \phi_1}{-\phi_2} + \frac{(\phi_1 + \phi_3 + \phi_4)}{-\phi_2} \log \phi_1 + \frac{\phi_4}{\phi_2} \log \mu \tag{16}
\]

which, together with our standard assumptions on the sign of \( \phi_4 \) and \( \phi_2 \)

\(^6\)Equation (12) implies that \( q_t(h) = 0 \).
implies that the elasticity of the unemployment rate with respect to inflation is negative

\[
\frac{d \log u}{d \log \mu} = \frac{\phi_4}{\phi_2} < 0
\] (17)

The intuition underlying this result is the following. An increase in inflation produces a decrease in the reference wage, by reducing the current real value of the past compensation. This would spur effort, but the firms’ optimal level of effort does not depend on inflation. As a consequence firms increase employment (and decrease unemployment) to keep the level of effort constant. Following the results by Karanassou et al. (2005, 2008a, 2008b), one could calibrate \( \frac{\phi_4}{\phi_2} \approx -0.29 \).

Note that this mechanism does not imply that hyperinflation will produce large decreases in unemployment. In order to understand this point we focus on the semielasticity of the unemployment rate with respect to the money growth rate. In our context, the advantage of the semi-elasticity versus the elasticity is that it is a measure of the reactiveness of the unemployment rate to absolute, and not percentage, changes in the money growth rate, mirroring, under this respect, the results provided by, among others, Ascari (1998, 2004) and Graham and Snower (2004, 2008). The semielasticity of the unemployment rate with respect to money growth is

\[
\frac{d \log u}{d \mu} = \frac{\phi_4}{\phi_2} \frac{1}{\mu} < 0
\] (18)

which is still negative, given that \( \mu \geq 1 \), but \( \lim_{\mu \to \infty} \frac{d \log u}{d \mu} = 0 \).

### 3.2 The short-run

In order to analyze the short run dynamics of the present economic model, consider first that the only steady state condition we imposed to obtain (16) is the equality of money growth and inflation. Out of steady state one can write (16) as \( \log u_{t+i} = \frac{\phi_0 - \phi_1}{\phi_2} + \frac{(\phi_3 + \phi_4 + \phi_1)}{\phi_2} \log \phi_1 + \frac{\phi_4}{\phi_2} \log \pi_{t+i} \). The other equations
of the system are (9), (10), the aggregate resource constraint, \( y_t = c_t \), the production function and the condition \( n_t = 1 - u_t \). The equilibrium for this model is a sequence \( \{ u_{t+i}, \pi_{t+i}, n_{t+i}, y_{t+i}, l_{t+i}, c_{t+i} \} \) satisfying households’ utility maximization and firms’ profit maximization.

This system of equations, after log-linearization around the steady state, can be expressed as a second order difference equation in inflation, which in its turn can be re-arranged to obtain the following system of first order difference equations

\[
E(\hat{x}_{t+i+1}) = \left[ t_{ss} + \frac{u_{ss}}{n_{ss}} \phi_\pi \right] \hat{x}_{t+i} - \frac{u_{ss}}{n_{ss}} \phi_\pi t_{ss} \hat{\pi}_{t+i} \tag{19}
\]

\[
E(\hat{\pi}_{t+i+1}) = \hat{x}_{t+i} \tag{20}
\]

In the equations above, hats denote deviations from steady state, \( \phi_\pi \equiv -\frac{\phi_1}{\phi_2} \) and \( i_{ss}, u_{ss} \) and \( n_{ss} \) are the steady state values of the nominal interest rate, of the unemployment rate and of the employment rate respectively. In order to investigate the stability of (19)-(20) we need to calibrate not only \( \phi_2 \) as above, but also \( i_{ss}, u_{ss} \) and \( n_{ss} \). In order to do so we take as reference the averages of the post-second-world-war US time series and we set \( u_{ss} = 0.056 \), \( n_{ss} = 1 - u_{ss} \) and \( i_{ss} = 1.02 \times (1 + \mu) \). We compute the roots of (19)-(20) for various values of trend inflation and the results are showed in Figure 1. As it is possible to see the system is always saddle-path stable, give that one root is outside the unit circle and the other one within it.

It is possible to wonder what are the effects of trend inflation on the stable arm of the system. The answer to this question is showed in Figure 2 where, following Shone (2001), different trajectories along the stable arm are projected on the \( \{ \pi_t, \pi_{t+1} \} \) plane for trend inflation rates equal to 2\%, 20\% and 80\%. The higher is trend inflation and the flatter is the stable arm. In other words, the higher is trend inflation and the sharper should inflation reductions be in order to achieve stability.
4 Second variation: the personal norm case

In the personal norm case, firms recognize that wage setting has intertemporal consequences. A wage increase will induce more effort in the first period by raising the household’s real wage, but it will decrease effort in the second period by raising the household’s reference wage. The firms’ profit maximization problem will therefore be

\[ \max_{\{n_{t+i}(h), W_{t+i}(h)\}} \sum_{j=0}^{\infty} \Delta_{t,t+i} \left[ P_{t+i}y_{t+i} - \int_{h=0}^{1} W_{t+i}(h)n_{t+i}(h)dh \right] \]

s.t. \( y_{t+i} = \left[ \int_{0}^{1} e_{t+i}(h) \frac{\partial W_{t+i}(h)}{\partial n_{t+i}} dh \right] \frac{\partial \epsilon_{t+i}}{\partial y_{t+i}} \) (21)

where \( \Delta_{t,t+i} \) is the firm discount factor.

In the present setting (13) turns out to be

\[ \Delta_{t,t+i}n_{t+i}(h) = \Delta_{t,t+i} \frac{y_{t+i}}{e_{t+i}} \left( \frac{\phi_{1}}{W_{t+i}(h)} \right) + E \left[ \Delta_{t,t+i+1} \frac{y_{t+i+1}}{e_{t+i+1}} \left( \frac{\phi_{4}}{W_{t+i}(h)} \right) \right] \] (22)

In words, firms equate the discounted marginal cost of increasing the real wage to the sum of its discounted marginal revenues, which are composed by a positive effort effect in period \( t+i \) and a negative effort effect in period \( t+i+1 \).

Consider that households and firms have access to a complete set of frictionless security markets, which, after Lucas (1978) and Collard and de la Croix (2000), implies that, at equilibrium, \( \Delta_{t,t+i} \) will be proportional to the discounted marginal value of wealth, which, assuming a logarithmic separable utility function in consumption and knowing that \( c_{t+i} = y_{t+i} \), will be
equal to $\beta^{t+i}/y_{t+i}$.

Substituting (12) — which holds also for the present model - into the previous equation and re-arranging one has

$$1 = \frac{\phi_1}{e_{t+i}(h)} + \frac{\beta}{e_{t+i+1}(h)}[\phi_4 \mu_{t+i+1}]$$

In steady state this implies a modified Solow condition, which, after dropping the $h$ index due to symmetry, is

$$e = \phi_1 + \beta \phi_4 \mu$$

Firms still demand the same effort level to all households across time, but not independently from money growth, given that they now take into account its discounted future effect on effort. As a consequence trend inflation appears to have a negative impact on firms’ desired level of effort. This happens because there are diminishing returns to the effort connected to $h - th$ kind of labour input. Under such circumstances trend inflation, equal to trend money growth, would induce households to elicit more effort in time $t+i+1$ by reducing the reference wage. However, under diminishing returns, this is less and less beneficial to firms and, as a consequence, the marginal revenue to wage increases would fall below their marginal cost. Firms, therefore, anticipate households’ behavior by demanding less effort to each household the greater is money growth. Due to symmetry, this produces a negative link between trend inflation and aggregate effort\(^7\).

This implies that the effect of money growth on unemployment does not vanish at high inflation rates. Along the lines followed in the previous section

\(^7\)A graphical account of this intuition is set out in Figure A1 in the Appendix, where (23) is depicted. The left hand side of (23) is the marginal cost of rising wages per unit of discounted labour. The right hand side, instead, is the marginal benefit, which is a decreasing function of the effort level because there are diminishing returns to the effort elicited by household $h$. Money growth reduces the marginal revenue to rising wages and therefore shifts inward the marginal revenue schedule, producing a fall in the desired level of effort, that balances the marginal revenue and cost to a wage increase.
it is easy to show that in the present model (18) turns out to be

$$\frac{d \log u}{d \mu} = \frac{\beta \phi_4}{\phi_2} - \frac{(\phi_1 + \phi_3 + \phi_4)}{\phi_2} \frac{\beta \phi_4}{(\phi_1 + \beta \phi_4 \mu)} + \frac{\phi_4}{\phi_2 \mu}$$

As a consequence \( \lim_{\mu \to \infty} \frac{d \log u}{d \mu} \neq 0 \), because firms hire more workers in the attempt to reduce effort as money growth rises. This is unrealistic and we will not develop the present model any further.

5 Third variation: the social norm case with wage staggering

5.1 The long-run

In the present section we combine efficiency wages with Taylor wage staggering. In order to do so we assume households to belong to different cohorts, whose labour services are not perfect substitutes. This assumption is necessary because if different labour kinds were perfect substitutes, labour demand for cohorts whose wage is reset would go to zero. The wage is not set by households, as usual in wage staggering model, but by firms, as customary in fair wages models.

Note that, due to the existence of wage staggering, households belonging to different cohorts have different income levels. However, as customary, we assume they have access to complete asset markets, which allows them to consume all the same amount of the final good as implied by the first order condition with respect to consumption in problem (1) – (2).

Following Graham and Snower (2004), one can write the firms’ profit
maximization problem as follows

$$\max_{\{n_{t+i}(h), W_{t+N_j}(h)\}} \sum_{j=0}^{\infty} \sum_{i=j}^{N-1} \Delta_{t,t+i} \left[ y_{t+i} - \int_{h=0}^{1} \frac{W_{t+N_j}(h)}{P_{t+i}} n_{t+i}(h) \, dh \right]$$

s.t. \( y_{t+i} = \left[ \int_0^1 e_{t+i}(h) \frac{a_{n-1}}{a_n} n_{t+i}(h) \frac{a_{n-1}}{a_n} \, dh \right]^{\frac{a_n}{a_{n-1}}} \) \( (26) \)

\( e(h) = \phi_0 + \phi_1 \log \frac{W_{t+N_j}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}} \)

where \( N \) is the contract length. The first order conditions with respect to \( n_{t+j}(h) \) and \( W_{t+N_j}(h) \) and the recursiveness of the problem above imply

$$\frac{W_i(h)}{P_{t+i}} = y_{t+i}^{\frac{1}{a_n}} e_{t+i}(h) \frac{a_{n-1}}{a_n} n_{t+i}(h)^{\frac{1}{a_n}} \quad (27)$$

$$\sum_{i=0}^{N-1} \Delta_{t,t+i} n_{t+i}(h) = \sum_{i=0}^{N-1} \Delta_{t,t+i} \left[ \int_0^1 e_{t+i}(h) \frac{a_{n-1}}{a_n} n_{t+i}(h) \frac{a_{n-1}}{a_n} \, dh \right]^{\frac{a_n}{a_{n-1}}} \cdot (28)$$

Substituting (27) into (28) one obtains

$$\sum_{i=0}^{N-1} \Delta_{t,t+i} n_{t+i}(h) \frac{a_{n-1}}{a_n} - \sum_{i=0}^{N-1} \Delta_{t,t+i} \frac{a_{n-1}}{a_n} \left[ \frac{\phi_1}{W_i(h)/P_{t+i}} \right] = 0 \quad (29)$$

which, given that \( \Delta_{t,t+i}, n_{t+i}(h), P_{t+i} > 0 \), leads to the Solow condition

$$e_{t+i}(h) = \phi_1 \quad (30)$$
Substituting (30) into \( y_{t+i} = \left[ \int_0^1 e_{t+i}(h) \frac{\theta_{n-1}}{\theta_n} n_{t+i}(h) \frac{\theta_{n-1}}{\theta_n} \, dh \right]^{\frac{\theta_n}{\theta_n-1}} \) one has

\[
1 = \frac{1}{\phi_1} \left\{ \int_0^1 \left[ \frac{W_t(h)}{P_{t+i}} \right]^{1-\theta_n} \, dh \right\}^{\frac{1}{1-\theta_n}}
\]

(31)

and in steady state

\[
\frac{W^*}{P} = \phi_1 \left[ \frac{1}{N} \frac{1 - \mu^{N(\theta_n-1)}}{1 - \mu^{\theta_n-1}} \right]^{\frac{1}{\theta_n-1}}
\]

(32)

where \( W^* \) is the reset wage.

Further, substitute the Solow condition into (7) and aggregate across households keeping in mind that \( \frac{P_{t+i}}{P_{t+i-1}} = \mu \) to obtain

\[
\phi_1 = \phi_0 + \phi_1 \sum_{j=0}^{N-1} \frac{\log \left( \frac{W^* P^{-j}}{N} \right)}{\phi_2} + \phi_2 \log \mu + (\phi_3 + \phi_4) \log \frac{W}{P} - \phi_4 \log \mu
\]

(33)

and

\[
\log u_{WS} = \frac{\phi_1 - \phi_0}{\phi_2} - \frac{\phi_1}{\phi_2} \log \left\{ \phi_1 \left[ \frac{1}{N} \frac{1 - \mu^{N(\theta_n-1)}}{1 - \mu^{\theta_n-1}} \right]^{\frac{1}{\theta_n-1}} \right\} + \frac{\phi_1 (N - 1)}{2} \log \mu + \frac{\phi_4}{\phi_2} \log \mu - \frac{(\phi_3 + \phi_4)}{\phi_2} \log \frac{W}{P}
\]

(34)

where the subscript \( WS \) stays for wage-staggering.

Subtracting (34) from (16) and taking the first order derivative with respect to \( \mu \), one can compute the semielasticity of the percentage deviation of the unemployment rate under wage staggering from its level with flexible
\[ \frac{\partial (\log u_{WS} - \log u)}{\partial \mu} = -\frac{\phi_1}{-\phi_2} \left[ \frac{N}{1 - \mu^{N(\theta_n-1)}} \right] \mu^{N(\theta_n-1)-1} + \frac{1}{-\phi_2} \left[ \frac{1}{1 - \mu^{(\theta_n-1)}} \right] \mu^{(\theta_n-1)-1} - \frac{\phi_1}{-\phi_2} \frac{(N-1)}{2} \frac{1}{\mu} \] (35)

If \( \frac{\partial (\log u_{WS} - \log u_{ff})}{\partial \mu} \) is negative, it will mean that unemployment will be more responsive to absolute changes in money growth under wage staggering than under flexible wages. In order to explore this issue, it is necessary to check that the following condition holds:

\[ \Omega(\mu) = -\frac{N}{[1 - \mu^{N(\theta_n-1)}]} \mu^{N(\theta_n-1)} + \frac{1}{[1 - \mu^{(\theta_n-1)}]} \mu^{(\theta_n-1)} - \frac{(N-1)}{2} > 0 \] (36)

We do so for different values of \( N \) and \( \theta_n \) in Figures 3 and 4 respectively. In both the cases (36) is verified.

The intuition for this result is that wage staggering has two effects on effort. On the one hand, wage dispersion increases with inflation, leading to a higher ratio between the wage of the resetting cohort and the aggregate wage index. On the other hand, a higher inflation rate means that, over the contract period, the real wage of not-resetting cohorts will decline faster. The former effect has a positive impact on effort, while the latter a negative one. However, the former prevails on the latter one. As a matter of consequence firms have to increase employment and decrease unemployment to a greater extent than under flexible wages in order to keep effort at their constant desired level. Increasing \( N \) and \( \theta_n \) boosts wage dispersion, decreasing the slope of the long-run Phillips curve.

\[ \text{Recall that } \frac{\phi_1}{\phi_2} < 0. \]
5.2 The short run

In order to analyze the short run dynamics of the present economic model, we set \( N = 2 \). The equation for the log of the unemployment rate can be obtained integrating the effort function over \( h \) and keeping in mind equation (31):

\[
\log u_{t+i} = \frac{\phi_0 - \phi_1}{-\phi_2} + \frac{\phi_1}{-\phi_2} \int_0^{1/2} \log \frac{W_{t+i}(h)}{P_{t+i}} \, dh + \frac{\phi_1}{-\phi_2} \int_{1/2}^{1} \log \frac{W_{t+i-1}(h)}{P_{t+i-1} \pi_{t+i}} \, dh - \frac{\phi_4}{-\phi_2} \log \pi_{t+i}
\]

The other equations of the system are (9), (10), (31), the aggregate resource constraint - \( y_t = c_t \) -, the definition of unemployment rate \( \int_0^{1/2} n_t(h) \, dh + \int_{1/2}^{1} n_t(h) \, dh = 1 - u_t \), and the demands for the labour services of the households belonging to the two cohorts:

\[
\frac{W_{t+i}(h)}{P_{t+i}} = \left[ \frac{y_{t+i}}{n_{t+i}(h)} \right]^{\frac{1}{\sigma_w}} \quad \text{for} \quad h \in \left[ 0, \frac{1}{2} \right]
\]

\[
\frac{W_{t+i-1}(h)}{P_{t+i-1} \pi_{t+i}} = \left[ \frac{y_{t+i}}{n_{t+i}(h)} \right]^{\frac{1}{\sigma_w}} \quad \text{for} \quad h \in \left[ \frac{1}{2}, 1 \right]
\]

Finally, the autoregressive process for money growth is

\[
\mu_t = \mu^{1-\zeta} \mu_{t-1}^{\zeta} \exp(\epsilon_t)
\]

The equilibrium for this model is a sequence \( \left\{ \frac{W_{t+i}(h)}{P_{t+i}}, \mu_{t+i}, u_{t+i}, \pi_{t+i}, n_{t+i}(0), n_{t+i}(1), y_{t+i}, u_{t+i}, c_{t+i} \right\} \) satisfying households’ utility maximization and firms’ profit maximization. We log-linearized the system around a steady state with \( u_{ss} = 0.056 \) on the basis of the US post-WWII experience. We calibrated the system parameters as customary in the New-Keynesian literature (see for instance Ascari, 2004): \( \beta = 1.04^{\frac{1}{2}}, \mu = 1.02^{\frac{1}{2}}, \theta_n = 5, \frac{\phi_4}{\phi_2} = 0.29, \zeta = 0.57^{\frac{1}{2}} \). In order to attach a value to \( \frac{\phi_4}{\phi_2} \) we note that it can be considered as the inverse of the elasticity of households’ wages with respect.
to the unemployment rate and so we set it to $0.07^{-1}$ after Nijkamp and Poot (2005).

Figure 5, as similar figures below, plots the percentage deviations from steady state of the inflation rate against those of the unemployment rate. In other words, we plot the impulse response function of the inflation rate against that of the unemployment rate in order to show the unemployment-inflation trade-off in a more direct way. As it is possible to see, wage staggering implies a flatter Phillips curve than flexible wages not only in the long-run but in the short run too. Note that increasing $\theta_n$ from 5 to 15 would not change our results markedly\textsuperscript{9}. Instead, increasing N from 2 to 4 has a considerable impact on the dynamics of inflation and unemployment. As showed in Figure 6, their reactivenss increases, however, unemployment first declines and then increases before going back to its steady state value. A shortcoming of this model is that, with difference to the other models presented in this work, a monetary expansion can cause a contraction in output due to the inefficiencies arising from firms shifting labour demand from one cohort to the other, given that different labour kinds are imperfect substitutes. For $N=4$ and $\theta_n = 5$ a one percentage shock in money growth produces a 0.18 percent decline in output. This is implausible and for this reason the model presented in this section is not our preferred one.

\section{Fourth variation: the social norm case with varying capital}

Once considering varying capital within the model, we assume the existence of capital adjustment costs after Bernanke et al. (1999) and Gertler (2002). The households’ budget constraint changes to

\textsuperscript{9}Further results are available from the author upon request.
\[ c_{t+i}(h) = \frac{W_{t+i}(h)}{P_{t+i}} n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + \frac{M_{t+i-1}(h)}{P_{t+i}} - \frac{B_{t+i}(h)}{P_{t+i}} + \frac{B_{t+i-1}(h)}{P_{t+i}} \left( t_{t+i-1} + \frac{R_{t+i}}{P_{t+i}} K_{t+i}(h) - \frac{Q_{t+i}}{P_{t+i}} [K_{t+i}(h) - (1 - \delta) K_{t+i-1}(h)] + q_{t+i}(h) \right) \]  

(41)

where \( K_{t+i}(h) \) is the capital held by household \( h \), \( \delta \) is the capital depreciation rate, \( R_{t+i} \) is the capital rental rate and \( Q_{t+i} \) is the nominal Tobin’s \( q \). Furthermore, households maximize utility with respect to capital too and interacting the first order conditions for capital and consumption leads, under households’ symmetry, to the following equation

\[ E(c_{t+i+1}) = \frac{R_{t+i}}{P_{t+i}} E(c_{t+i+1}) + c_{t+i}^{1} \left( 1 - \delta \right) \frac{Q_{t+i+1}}{P_{t+i+1}} \]  

(42)

As in the New-Keynesian tradition, we assume the existence of an intermediate labour market, where labour intermediaries hire households’ horizontally differentiated labour inputs to produce homogeneous labour to be sold to firms operating on the final product market. In the intermediate labour market we assume productivity to depend on effort. The profit maximization problem of labour intermediaries is

\[
\begin{align*}
\max_{\{n_{t+i}(h), W_{t+i}(h)\}} & \quad W_{t+i} n_{t+i} - \int_0^1 W_{t+i}(h)n_{t+i}(h)dh \\
\text{s.t.} & \quad n_{t+i} = \left[ \int_0^1 \eta_{t+j}^{\frac{\sigma_n - 1}{\eta_n}} n_{t+j}(h) \frac{\sigma_n - 1}{\eta_n} dh \right]^{\frac{\eta_n}{\sigma_n - 1}} 
\end{align*}
\]  

(43)

The solution of this problem and households’ symmetry imply

\[ \frac{W_{t+i}(h)}{W_{t+i}} = \frac{n_{t+i}(h)}{n_{t+i}(h)} = \phi_1 = e_{t+i} = 1 \]  

(44)

Firms in the final product market maximize profits hiring labour and
capital and adopting a Cobb-Douglas production function. The solution of their problem leads to two customary demand functions for labour and capital

\[(1 - \alpha) \frac{y_{t+i}}{W_{t+i}} = n_{t+i} \quad (45)\]
\[\alpha \frac{y_{t+i}}{P_{t+i}} = K_{t+i} \quad (46)\]

Substituting these two equations into the production function one has

\[\frac{W_{t+i}}{P_{t+i}} = \left( \frac{R_{t+i}}{P_{t+i}} \right)^{\frac{\alpha}{\alpha - 1}} (1 - \alpha) \quad (47)\]

Finally, capital producer \(j\) has the following production function

\[Y_{t+i}^k (j) = \phi \left[ \frac{I_{t+i}^j (j)}{K_{t+i-1}^j (j)} \right] K_{t+i} (j) \quad (48)\]

where \(Y_{t+i}^k (j)\) is new capital, \(I_{t+i}^j (j)\) is raw output used as material input at time \(t + i\) and \(\phi' (\cdot) > 0, \phi'' (\cdot) < 0, \phi (0) = 0\) and \(\phi \left( \frac{I}{K} \right) = \frac{I}{K}\), with \(\frac{I}{K}\) being the steady state investment-capital ratio. \(K_{t+i} (j)\) is capital rented after it has been used to produce final output within the period. The profits of the \(j\)-th capital producer can be written as \(\frac{Q_{t+i}}{P_{t+i}} \phi \left[ \frac{I_{t+i}^j (j)}{K_{t+i-1}^j (j)} \right] K_{t+i} (j) - I_{t+i}^j (j) - Z_{t+i}^k K_{t+i} (j)\) where \(Z_{t+i}^k\) is the rental price of capital used for producing new capital. The first order condition for \(I_{t+i}^j (j)\) is, under a symmetry condition:

\[\frac{Q_{t+i}}{P_{t+i}} \phi' \left( \frac{I_{t+i}^j}{K_{t+i-1}} \right) - 1 = 0 \quad (49)\]

where \(I_{t+i} = \int_0^1 I_{t+i}^j (j) \, dj\) and \(K_{t+i-1} = \int_0^1 K_{t+i-1}^j (j) \, dj\). One can show that the first order condition with respect to \(K_{t+i} (j)\), \(\phi \left( \frac{I}{K} \right) = \frac{I}{K}\) and (49) imply that \(Z_{t+i}^k\) is approximately zero near the steady state and so it can be ignored.
The system of equations is therefore composed by (9), (10), the aggregate resource constraint \( y_{t+i} = c_{t+i} + I_{t+i} \), the law of motion of capital \( K_{t+i} = \phi_K \left( \frac{I_{t+i}}{K_{t+i-1}} \right) K_{t+i-1} - (1 - \delta) K_{t+i-1} \), the definition of the unemployment rate \( n_t = 1 - u_t \), (40), (42), (45), (46), (47), (49) and (7), which imposing (44) and after rearranging becomes

\[
\log u_{t+i} = \frac{\phi_0 - \phi_1}{-\phi_2} - \frac{\phi_4}{\phi_2} \log \pi_{t+i} + \frac{\phi_1 + \phi_3}{-\phi_2} \log \frac{W_{t+i}}{P_{t+i+1}} + \frac{\phi_4}{\phi_2} \log \frac{W_{t+i+1}}{P_{t+i+1}} \tag{50}
\]

The equilibrium of this system is a sequence \( \{ R_{t+i}, \frac{W_{t+i}}{P_{t+i}}, y_{t+i}, n_{t+i}, K_{t+i}, c_{t+i}, u_{t+i}, \pi_{t+i}, t_{t+i}, I_{t+i}, \frac{Q_{t+i}}{P_{t+i}} \} \) satisfying utility and profit maximization problems.

Regarding the long-run we note that in steady state the real Tobin’s q is equal to one and therefore that \( \frac{R}{P} \) and \( \frac{W}{P} \) are pinned down by (42) and (47) independently from money growth. On the basis of (50) and of the steady state equality of inflation and money growth, this entails that (17) and (18) also hold for the present model.

Regarding the short-run, we do not change the calibration of the parameters that already appeared in the previous sections of the present work, with the only exception that, given that we have flexible wages here, we do not rise them to the power of \( \frac{1}{2} \). Following the same reasoning above regarding the elasticity of the wage to the unemployment rate we set \( \frac{\phi_1 + \phi_3}{\phi_2} = 0.07^{-1} \). Furthermore, as customary, \( \alpha = 0.33, \delta = 1 - 0.92 \) and, after Bernanke et al. (1999), \( \eta = -\frac{\phi_1}{\phi_2} \frac{\pi}{\pi} = 0.5 \). We log-linearize the system around the steady state. The short-run Phillips curve with fixed and varying capital are plotted in Figure 7. The result that higher inflation goes hand in hand with a lower unemployment rate, whose intuition was discussed commenting equation (17), is confirmed also for the present model. As it is possible to see, varying capital implies a flatter short run Phillips curve than under fixed capital, given that the boom following a monetary expansion is reinforced by
an increase of investments, which rise upon impact by 0.08%\textsuperscript{10}. Figure 7 also shows that increasing trend inflation decreases the responsiveness of both the inflation and unemployment rates to a 1\% monetary shock. This is consistent with our results above that increasing trend inflation flattens the stable arm of the economic system without capital and it can be explained by keeping in mind two facts. First, households smooth consumption and, second, an increase in trend inflation decreases the elasticity of the money demand function to the nominal interest rate\textsuperscript{11}. If households smooth consumption, they will tend to smooth also real money holdings - see equation (10). This, in presence of a smaller reactiveness of money demand to the nominal interest rate, can happen only thanks to a larger reaction in the latter one (Figure 8). In other words, households achieve a stable path for consumption and real money holdings in face of a monetary shock with higher trend inflation by letting the interest rate to react more, which stabilizes the whole economy and implies a smaller change in inflation too. A smaller change in the inflation rate translates into a smaller change in the unemployment rate via the Phillips curve (50).

\textsuperscript{10}Changing \( \eta \) would only have negligible effects on the Phillips curve. Further results are available from the author on request. It is worth noting that our model does not produce a persistent reaction of either the unemployment or the inflation rate after a monetary shock. This accords well with the empirical evidence produced by the inflation persistence network, whose main result is that, once allowing for structural breaks in the mean of the inflation time series, inflation has low persistence (Altissimo et al., 2007). Empirical evidence of a fast adjustment of unemployment after a monetary shock was produced by Karanassou et al. (2007, p. 346) where the unemployment rate takes just two periods to hit its new long-run level after a permanent monetary shock. However, this low persistence is not a property of efficiency wages themselves. Danthine and Kurmann (2004, 2010) showed that, once efficiency wages are coupled with price rigidities, it is possible to produce persistent impulse response functions.

\textsuperscript{11}Loglinearizing (10), one can show that this elasticity is \( \frac{1}{\bar{t}_{ss} - 1} \) where \( i_{ss} \) is equal to trend inflation over the discount factor.
7 Fifth and sixth variations: reciprocity in labor relations and the Phillips curve

The present section adopts an approach à la Danthine and Kurmann (2008, 2010), that can be nested into our model by specifying \( G[e_{t+i}(h)] = \frac{1}{2} [e_{t+i}(h)]^2 - \mathcal{R}[e_{t+i}(h), .] \), where \( \mathcal{R}[e_{t+i}(h), .] \) is the product of the gifts of the representative worker, \( d[e_{t+i}(h), .] \), and the firm \( g[W_{t+i}(h), .] \). In words, when perceiving a generous wage offer by the firm - \( g[W_{t+i}(h), .] > 0 \) - the utility of a worker increases by eliciting more effort - \( d[e_{t+i}(h), .] > 0 \). Note that 
\[
d[e_{t+i}(h), .] = [e_{t+i}(h)]^\zeta \quad \text{with} \quad 0 < \zeta < 1
\]
d in the above equation, \( \log \frac{W_{t+i}(h)}{P_{t+i}} \) accounts for the consumption utility attached by the representative worker to the firm’s actual wage offer. \( \log \frac{y_{t+i}}{n_{t+i}(h)} \) proxies for firms’ ability to pay, by describing the utility obtained if the firm distributed its whole revenue to workers. In case a worker quits and finds a job elsewhere, s/he will enjoy the expected utility \( \log \frac{w_{t+i}}{n_{t+i}(h)} \). Finally, 
\[
\log \left\{ (1 - s) \left[ \frac{W_{t+i-1}(h)}{P_{t+i}} \right] + s \left( \frac{W_{t+i-1}}{P_{t+i}} \right) \right\}
\]
represents the effect of the current real value of past compensation on the reference wage. This formulation encompasses both the social norm case (with \( s = 1 \)) and the personal norm one (with \( s = 0 \)). Finally, \( f_1, f_2 \) and \( f_3 \) are non-negative parameters.

The condition \( G'[e_{t+i}(h)] = 0 \) here implies the following effort function
\[
e_t = \frac{1}{\frac{1}{2} - \zeta} \left( \log \frac{W_{t+i}(h)}{P_{t+i}} - f_1 \log \frac{y_{t+i}}{n_{t+i}(h)} - f_2 \log \frac{W_{t+i}}{P_{t+i}} n_{t+i} - f_3 \log \left\{ (1 - s) \left[ \frac{W_{t+i-1}(h)}{P_{t+i}} \right] + s \left( \frac{W_{t+i-1}}{P_{t+i}} \right) \right\} \right)^{\frac{1}{2-\zeta}} (52)
\]

We perform the firm profit maximization problem as in our second vari-
ation above to obtain the first order conditions with respect to $W_{t+i}(h)$ and $n_{t+i}(h)$. Under household symmetry they are

$$\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} \left[ 1 + e_{t+i}^{\zeta-2} \frac{1}{2-\zeta} \zeta f_1 \right]$$  \hspace{1cm} (53)

$$\Delta_{t,t+i} n_{t+i} = \left\{ \begin{array}{l}
\Delta t_{t+i}, \frac{y_{t+i}}{e_{t+i}} \left( \frac{1}{2-\zeta} e_{t+i}^{\zeta-1} \frac{\zeta}{P_{t+i}} \right) - \\
-\Delta_{t,t+i+1} \frac{y_{t+i+1}}{e_{t+i+1}} \left[ \frac{1}{2-\zeta} e_{t+i+1}^{\zeta-1} \frac{\zeta f_3}{P_{t+i+1}} \left( \frac{1}{1-s} \right) \right]
\end{array} \right\}$$ \hspace{1cm} (54)

After some manipulation and interacting (53) and (54), one obtains a similar steady state equation to (24):

$$e = \left\{ \frac{1}{2-\zeta} \left[ 1 - \beta \left( \frac{f_3 \mu}{1-s} \right) - f_1 \right] \right\}^{1/\gamma}$$ \hspace{1cm} (55)

Along the lines followed in the previous variations it is possible to show that $\lim_{\mu \to \infty} \frac{\partial \log n}{\partial \mu} \neq 0$. In other words, one obtains the same result as in our second variation at the price of a heavier parametrization.

We now focus on the social norm case, namely we impose $s = 1$. Under this assumption (54) and (55) change into

$$n_{t+i} = \frac{\zeta}{2-\zeta} \frac{y_{t+i} e_{t+i}^{\zeta-2}}{P_{t+i}}$$ \hspace{1cm} (56)

$$e_{t+i} = \left[ (1 - f_1) \frac{1}{2-\zeta} \right]^{1/\gamma}$$ \hspace{1cm} (57)

On the footsteps of our first variation, it is possible to show that the employment rate and the inflation rate are linked by the following equation

$$\log n_{t+i} = \text{constant} + \frac{f_3}{f_2} \log \pi_{t+i}$$ \hspace{1cm} (58)

This has similar implications for the short-run and long-run Phillips curve to
those of our first variation at the additional cost of heavier parametrization.

8 Conclusions

In the present paper, we explored the relationship between inflation and unemployment in different models with fair wages and reciprocity in labor relations. We showed that, under customary assumptions regarding the parameters of the effort function, they have negative long- and short-run nexuses. This is motivated by the fact that firms respond to inflation - which spurs effort via a decrease in the reference wage - by increasing employment in order to maintain the effort level constant, as implied by the Solow condition. Under wage staggering this effect is stronger because wage dispersion magnifies the impact of inflation on effort. This effect is also stronger in the short-run once considering varying instead of fixed capital as booms generated by monetary expansions are reinforced by greater investment.

Once considering the personal norm case, the model produces an unrealistic negative impact of hyper-inflation on unemployment. Furthermore, under wage-staggering the model can produce output contractions in response to monetary expansions. Finally, shifting to a model with reciprocity in labour relations does not substantially change our results to the price of a much heavier parametrization. For these reasons, our preferred variation is the social norm case with flexible wages and, possibly, varying capital.

Our results can offer new theoretical insights into the evidence produced by recent empirical contributions finding a negative long-run relationship between unemployment and inflation.

References


Figure 1 – The roots of the system for different trend inflation rates
Figure 2 - The stable arm for different trend inflation rates

- trend inflation=2% per year
- trend inflation=20% per year
- trend inflation=80% per year

-0.005 0 0.005 0.01 0.015 0.02 0.025 0.03 0.035 0.04

inflation % deviation from steady state at time t+1

-0.2 0 0.2 0.4 0.6 0.8 1 1.2

inflation % deviation from steady state at time t

trend inflation=2% per year
trend inflation=20% per year
trend inflation=80% per year
Notes: $\theta_n$ was set equal to 5; for a definition of $\Omega(\mu)$ see equation (22).
Figure 4 – $\Omega(\mu)$ for different money growth rates and elasticities of substitution among labour kinds

Notes: N was set equal to 2; for a definition of $\Omega(\mu)$ see equation (22).
Figure 5 – The short-run Phillips curve with flexible and staggered wages
Figure 6 – The short-run Phillips curve with staggered wages and with different number of cohorts

The graph shows the short-run Phillips curve with staggered wages and for two different numbers of cohorts: two cohorts and four cohorts. The x-axis represents the percentage deviation of the unemployment rate from the steady state, while the y-axis represents the percentage deviation of the inflation rate from the steady state.
Figure 7 – The short-run Phillips curve with fixed and varying capital

- fixed capital
- varying capital and 2% trend inflation
- varying capital and 20% trend inflation

percentage deviation of the inflation rate from steady state

percentage deviation of the unemployment rate from steady state

-0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.5 1 1.5 2 2.5
Figure 8 – Impulse response function of the nominal interest rate after a 1% monetary shock under different trend inflation rates
Figure A1 - Firms equating marginal cost and revenues to wage increases under efficiency wages and diminishing returns to effort of different labour kinds

Marginal cost/revenue to a wage increase per unit of discounted labour

Marginal cost schedule

Marginal revenue schedule for $\mu = \mu_0$

Marginal revenue schedule for $\mu = \mu_1 > \mu_0$

Firms' desired level of effort