Budgeting Models and System Simulation: a dynamic approach

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Introduction

The main idea is to create a flexible Budgeting Model which works in a dynamic framework capable of providing information about the financial position, income, assets and liabilities of an enterprise. This new tool should be considered the starting point for a new class of models able to adapt to the new informative requirements that come from the different economic subjects. In order to create a similar model one has to analyze the procedures of double entry bookkeeping and find a mathematical formalization of them.
Introduction

During the last five decades many economists and accountants have worked to give a mathematical formalization to bookkeeping double entry and more in general an axiomatization of accounting procedures;

The attempts of finding a mathematical formalization has not led to a complete defined framework;

Thus in literature we can distinguish many different frameworks where an accounting axiomatization has been utilized;
The aim of our studies will be to gather previous literature results formalizing a model able to provide some information about the financial position, income, asset and liabilities of an enterprise in a framework utterly dynamic.
This paper is mainly based on the following previous literature results:

1. Mattessich’s articles, “Budgeting Model and System Simulation” (1961) and “Mathematical Models in Business Accounting” (1958). In these papers the main idea is to create an a flexible budgeting system, using a set of simultaneous equation, that give to the management the opportunity to measure different management policies or different options.


In this presentation you are going to use a balance sheet as similar as used by Mattessich (1961) with no contingent claims.

1. $L_n$ Cash and Cash equivalence;
2. $C_n$ Trade receivables;
3. $R_n$ Inventories;
4. $K_n$ Property, Plan and Equipment;
5. $D_n$ Trade and other payables;
Assumptions

- Constant amortization schedule;
- Property, Plant and Equipment in every period will be net of accumulation depreciation expense;
- Long debts and Mortgages have to pay obligations;
- Cash and Cash equivalence are deposited on a bank account with a credit interest;
- In our model you will consider just unsold stock and the authors have decided to evaluate them at Weighted Average Cost;
- Trade receivables will be net of fund containing bad credits, or credits we are not sure they will be settled.
A Variable Dynamic Model

\[
\begin{align*}
L_n &= (1 + \beta) \cdot L_{(n-1)} + \bar{\eta}_n \cdot (C_{1(n-1)} + F_n) + \gamma_n \cdot K_{(n-1)} - \bar{\omega}_n \cdot (D_{(n-1)} + G_n) \\
&\quad - d_n \cdot Df_{(n-1)} + (n \cdot F_n - m \cdot r \cdot n) + p/m_n \\
C_n &= (1 - \bar{\eta}_n) \cdot (C_{(n-1)} + F_n) \\
R_n &= \frac{V_n}{V_{n-1}} \cdot R_{(n-1)} + V_n \cdot (Y'_{n} - Y_n) \\
K_n &= (1 - \gamma_n) \cdot K_{(n-1)} - A_n \\
D_n &= (1 - \bar{\omega}_n) \cdot (D_{(n-1)} + G_n) \\
Df_n &= Df_{(n-1)} + (n \cdot F_n - m \cdot r \cdot n)
\end{align*}
\]
A Variable Dynamic Model

Some specifications:
- initial condition

\[
\begin{align*}
L_{n-1} &= \ldots \\
C_{n-1} &= \ldots \\
R_{n-1} &= \ldots \\
K_{n-1} &= \ldots \\
D_n &= \ldots \\
D_f_n &= \ldots \\
\end{align*}
\]

- \( \bar{\eta} = \frac{\eta_1 \cdot C_{n-1} + \eta_2 \cdot F_n}{C_{n-1} + F_n} \) and \( \bar{\omega} = \frac{\omega_1 \cdot D_{n-1} + \omega_2 \cdot G_n}{D_{n-1} + G_n} \)
Matrix Representation

\[
\begin{bmatrix}
L_n & C_n & R_n & K_n & D_n & Df_n \\
1 + \beta_n & \eta_n & 0 & \gamma_n & -\omega_n & -d_n \\
0 & 1 - \eta_n & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{V_n}{V_{n-1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 - \gamma_n & 0 & 0 \\
0 & 0 & 0 & 0 & 1 - \omega_n & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
L_{n-1} & C_{n-1} & R_{n-1} & K_{n-1} & D_{n-1} & Df_{n-1} \\
0 & 0 & (\Delta_n)V_{n-1} & 0 & 0 & 0 \\
\end{bmatrix}
+ \begin{bmatrix}
\Delta f_n + \psi_n \\
\end{bmatrix}
\]

- \( \Delta_n = Y'_n - Y_n \)
- \( \Delta f_n = n.F_n - m.r_n \)
- \( \psi_n = p/m_n \)
It is easy to see that others accounting items can be introduced, as:

- Financial instruments (Held for trading/Held to Maturity);
- Taxation;
- Others;
Example (Held to Maturity)

Held-to-maturity investments are financial assets with fixed or determinable payments and fixed maturity that an entity has the positive intention and ability to hold to maturity. Held-to-maturity investments are measured at amortised cost.

- $\alpha_n$ nominal interest rate;
- $(1 - \lambda_n)$ movements inside the accounting items;
- $\frac{P_n}{P_{n-1}}$ Amortised cost;
Example (Held to Maturity)

\[
\begin{bmatrix}
L_n \\
C_n \\
R_n \\
Kf_n \\
D_n \\
Df_n \\
\end{bmatrix}
= \\
\begin{bmatrix}
1 + \beta_1 \\
\eta_n \\
\alpha_n + \lambda_n \\
\Omega_n \\
-\omega_n \\
-d_n \\
\end{bmatrix}
\begin{bmatrix}
L_{n-1} \\
C_{n-1} \\
Kf_{n-1} \\
K_{n-1} \\
D_{n-1} \\
Df_{n-1} \\
\end{bmatrix}

\begin{bmatrix}
0 \\
F_n \\
0 \\
0 \\
G_n \\
0 \\
\end{bmatrix}
+ \\
\begin{bmatrix}
\Delta f_n + \psi_n \\
\Delta f_n \\
\end{bmatrix}
\]

- \( \Delta f_n = n.F_n - m.r.n \)
- \( \psi_n = p/m_n \)
No-Costant Model

A general solution for the no constant model is the following:

\[
\begin{align*}
S_0 &= M_n \cdot (S_{n-1} + CE_{n-1}) + F_n \\
S_0 &= S \\
S_1 &= M_1 \cdot (S_0 + CE_0) + F_1 \\
S_2 &= M_2 \cdot (M_1 S_0 + M_1 CE_0 + F_1 + CE_1) + F_2 \\
S_3 &= M_3 \cdot (M_2 M_1 S_0 + M_2 M_1 CE_0 + M_2 F_1 + M_2 CE_1 + F_2 + CE_2) + F_3 \\
S_4 &= M_4 M_3 M_2 M_1 S_0 + M_4 M_3 M_2 M_1 CE_0 + M_4 M_3 M_2 F_1 + M_4 M_3 M_2 CE_1 + M_4 M_3 M_2 CE_2 + F_3 \\
\vdots \\
S_n &= \prod_{h=n}^{1} M(h) \cdot (S_0 + CE_0) + \sum_{j=2}^{n} [(\prod_{h=n}^{j} M(h))(F_{j-1} + CE_{j-1})] + F_n
\end{align*}
\]
No-Costant Model

Hence, given the transition matrices $M(1)\ldots M(n) = \prod_{h=n}^{j} M(h)$, the vectors $CE1\ldots CEn$ and $F1\ldots Fn$, we can reach a formula that yields the cash flow and the whole balance sheet at $t = n$, defining in this way the balance sheet through a sequence no more recursive. However, looking the matrix you note that its shape is:

$$
\begin{bmatrix}
m_{11}(h) & m_{12}(h) & m_{13}(h) & m_{14}(h) & m_{15}(h) & m_{16}(h) \\
0 & m_{22}(h) & 0 & 0 & 0 & 0 \\
0 & 0 & m_{33}(h) & 0 & 0 & 0 \\
0 & 0 & 0 & m_{44}(h) & 0 & 0 \\
0 & 0 & 0 & 0 & m_{55}(h) & 0 \\
0 & 0 & 0 & 0 & 0 & m_{66}(h)
\end{bmatrix}
$$
No-Costant Model

It is easy to prove that \( M(1) \cdot M(2) \cdots M(n) = \prod_{h=1}^{n} M(h) \) will have of the same form of \( M \). Thus, if our goal is to write down a mathematical expression for each accounting items at time \( n \), it will be enough to evaluate every matrix’s element at time \( n \).

\[
\begin{bmatrix}
[\prod_{h=1}^{0} m_{11}(h)] & [m_{12}(0) \cdot m_{11}(1) + m_{12}(1) \cdot m_{22}(0)] & \cdots & [m_{16}(0) \cdot m_{11}(1) + m_{16}(1) \cdot m_{66}(0)] \\
0 & [\prod_{h=1}^{0} m_{22}(h)] & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 \\
0 & 0 & 0 & [\prod_{h=1}^{0} m_{66}(h)]
\end{bmatrix}
\]
In order to obtain a close formula, that gives us the balance sheet at $t = n$, you have to calculate the matrix’s elements, at time $n$, that compose the part $\prod_{h=n}^{1} M(h)$ and the elements that compose the part $\sum_{j=2}^{n}(\prod_{h=n}^{j} M(h))$ of the general solution.
Diagonal and First Row

Using mathematical induction is possible to demonstrate that the evolution for the elements of the first row and the elements the diagonal will be:

- diagonal \( n^{m_{ii}} = \prod_{h=n}^{1} m_{ii}(h) \);
- first row \( n^{m_{1i}} = \sum_{h=1}^{n} [m_{1j}(h) \cdot (\prod_{b=1}^{h-1} m_{jj}(b)) \cdot (\prod_{c=h+1}^{n} m_{11}(c))] \)

With the assumptions that:

- \( \prod_{b=1}^{h-1} m_{jj}(b) = 1 \) if \( h = 1 \)
- \( \prod_{c=h+1}^{n} m_{11}(c) = 1 \) if \( h = n \).
First Row and Diagonal

The next step is to give a mathematical shape to the last part of the dynamic solution, namely $\sum_{j=2}^{n}(\prod_{h=n}^{j} M(h))$. This is enough simple, because this is nothing else than a summation of product structures, so both for the element of first row and the diagonal, change only the superior index above the product structure.

With the assumptions that:

- $\prod_{b=1}^{h-1} m_{jj}(b) = 1$ if $h = i$
- $\prod_{c=h+1}^{n} m_{11}(c) = 1$ if $h < c$. 
A close formula for each Accounting Item

\[ L_n = \prod_{h=n}^{1}(1 + \beta(h)) \cdot L_0 + \sum_{h=1}^{n}[\eta(h) \cdot (\prod_{b=1}^{h-1}(1 - \eta(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)))] \cdot (C_0 + F_0) + \\
+ \sum_{h=1}^{n}[\gamma(h) \cdot (\prod_{b=1}^{h-1}(1 - \gamma(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)))] \cdot K_0 + \\
+ \sum_{h=1}^{n}[\omega(h) \cdot (\prod_{b=1}^{h-1}(1 - \omega(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c))) \cdot (D_0 + G_0) + \\
+ \sum_{h=1}^{n}[d(h) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)))] \cdot Df_0 + \\
+ \sum_{j=2}^{n} \cdot [\prod_{h=n}^{j}(1 + \beta(h))] \cdot (n.F_{j-1} - m.r_{j-1} + p/m_{j-1}) + \\
+ \sum_{h=j}^{n} \eta(h) \cdot (\prod_{b=1}^{h}(1 - \eta(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)) \cdot F_{j-1} + \\
+ \sum_{h=j}^{n} \gamma(h) \cdot (\prod_{b=1}^{h}(1 - \gamma(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)) \cdot A_{j-1} + \\
- \sum_{h=j}^{n} \omega(h) \cdot (\prod_{b=1}^{h}(1 - \omega(b))) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)) \cdot G_{j-1} + \\
- \sum_{h=j}^{n} d(h) \cdot (\prod_{c=h+1}^{n}(1 + \beta(c)) \cdot (n.F_{j-1} - m.r_{j-1})] + n.F_n - m.r_n \]
A close formula for each Accounting Item

\[ C_n = \prod_{h=n}^{1} (1 - \eta(h)) \cdot (C_0 + F_0) + \sum_{j=2}^{n} [\prod_{h=n}^{j} (1 - \eta(h))] \cdot F_{j-1} \]
\[ R_n = \prod_{h=n}^{1} (1 - \frac{V_h}{V_{h-1}}(h)) \cdot (R_0 + (Y_0' - Y_0) \cdot V_0) + \sum_{j=2}^{n} [\prod_{h=n}^{j} (1 - \frac{V_h}{V_{h-1}}(h))] \cdot (Y_{j-1}' - Y_{j-1}) \cdot V_{j-1} \]
\[ K_n = \prod_{h=n}^{1} (1 - \gamma(h)) \cdot D_0 + \sum_{j=2}^{n} [\prod_{h=n}^{j} (1 - \gamma(h))] \cdot A_{j-1} + A_n \]
\[ D_n = \prod_{h=n}^{1} (1 - \omega(h)) \cdot (D_0 + G_0) + \sum_{j=2}^{n} [\prod_{h=n}^{j} (1 - \omega(h))] \cdot G_{j-1} \]
\[ Df_n = Df_0 + \sum_{j=2}^{n} (n \cdot F_{j-1} - m \cdot r_{j-1}) + n \cdot F_n - m \cdot r_n \]
A model with all coefficients constant can be considered a particular case of the previous model. A similar model it has been already presented during the last seminar so we are going just specified the highlights to introduce our current researches. We are going to specify:

1. the constant model
2. the general close formula;
3. the entire model;
4. the averages;
The Constant Model

\[
L_n = L + \bar{\eta} \cdot (C_{n-1} + F_n) + \gamma \cdot K_{n-1} - \bar{\omega} \cdot (D_{n-1} + G) - d \cdot Df_{n-1} + (n.F - m.r + p/m)
\]

\[
C_n = (1 - \bar{\eta}) \cdot (C_1(n-1) + F)
\]

\[
R_n = V \cdot R_{n-1} + V \cdot (Y' - Y)
\]

\[
K_n = (1 - \gamma) \cdot K_{n-1} - A
\]

\[
D_n = (1 - \bar{\omega}) \cdot (D_{n-1} + G)
\]

\[
Df_n = Df_{n-1} + (n.F - m.r)
\]
The Close Formula

It easy to demonstrate that the close formula is just a particular case of the no-constant model. In fact if we consider all the matrix equals, then $M(1)\ldots M(n) = \prod_{h=n}^{1} M(h) = M^n$. Thus arranging the formula we obtain the following:

$$S_n = M^n \cdot S_0 + \sum_{l=1}^{n} M^l \cdot CE + \sum_{l=0}^{n-1} M^l \cdot F$$

We have just to point out that we assume that $\beta$ tent to zero, this means that we do no have credit interest.
The Entire Model

\[ L_n = L_0 + \left[ 1 - (1 - \eta_1)^n \right] \cdot C_{10} + \left[ 1 - (1 - \gamma)^n \right] \cdot K_0 - \left[ (1 - (1 - \omega)^n \right] \cdot D_{10} + \]
\[ d_n \cdot Df_0 + \left[ n - (1 - \eta) \left( \frac{1 - (1 - \eta)}{\eta} \right)^n \right] \cdot F - \left[ n - (1 - \omega) \left( \frac{1 - (1 - \omega)}{\omega} \right) \right] \cdot G + \]
\[ n \cdot (n.F - m.r + p/m) + \left[ n - \left( \frac{1 - (1 - \gamma)}{\gamma} \right) \right] \cdot A - d \left[ n \cdot \frac{(n-1)}{2} \right] \cdot (n.F - m.r) \]

\[ C_n = (1 - \eta)^n \cdot C_0 + (1 - \eta) \cdot \left[ \frac{1 - (1 - \eta)^n}{\eta} \right] \cdot F \]

\[ R_n = C_n = V^n \cdot R_0 + V \cdot \left[ \frac{1 - V^n}{1 - V} \right] \cdot (Y' - Y) \]

\[ K_n = (1 - \gamma)^n \cdot K_0 + \left[ \frac{1 - (1 - \gamma)^n}{\gamma} \right] \cdot A \]

\[ D_n = (1 - \omega)^n \cdot D_0 + (1 - \omega) \cdot \left[ \frac{1 - (1 - \omega_1)^n}{\omega_1} \right] \cdot G \]

\[ Df_n = Df_0 + n \cdot (n.F - m.r) \]
Averages

We have already shown, in the last seminar, the possibility to find a series of averages for every model’s coefficient, endogenous and exogenous, which are able to replicate the result of the model with variable parameters, after $n$ period. Here we make use the same technique, we just show the underline idea based on the concept of Chisini’s average.
Average’s idea

The substance of the average concept introduced by Chisini can be formalize in these form:

\[ f(x_1, x_2, \ldots, x_n) = f(x, x, \ldots, x) \quad (x \text{ for } n \text{ times}) \]

We are going to use the same concept, to discover a series of averages that permit of reproducing the values assumed from the model after \( n \) periods.
Rewriting the Cash flow formula $L_n$ in this way:

$$L_n = W_n + P_n + T_n + Z_n$$

where

$$W_n = [1 - (1 - \eta_1)^n] \cdot C_0 + \left[ n - (1 - \eta)\left(\frac{1-(1-\eta)}{\eta}\right)^n \right] \cdot F$$

$$P_n = [1 - (1 - \gamma)^n] \cdot K_0 + \left[ n - \left(\frac{1-(1-\gamma)}{\gamma}\right) \right] \cdot A$$

$$T_n = [(1 - (1 - \omega)^n] \cdot D_{10} + \left[ n - (1 - \omega)\left(\frac{1-(1-\omega)}{\omega}\right) \right] \cdot G$$

$$Z_n + \sum (p/m) =
\quad dn \cdot Df_0 + n \cdot (n.F - m.r + p/m) - d\left[\frac{n \cdot (n-1)}{2}\right] \cdot (n.F - m.r)$$
Analyzing the Cash flow formula $L_n$, it is of primary importance to note that not everything is a function of the whole set of variables, in fact we notice, for example, that just $W_n$ depend on $\eta$ and $F$ and looking the other equations is possible to realize that just $C_n$ is function of the same variables. Thus we can write down an equation system in order to find an average for coefficient $\eta$ and $F$. 
η and $F$

$$
\begin{align*}
W_n &= [1 - (1 - \eta_1)^n] \cdot C_{10} + \left[ n - (1 - \eta) \left( \frac{1 - (1 - \eta)}{\eta} \right)^n \right] \cdot F \\
C_n &= (1 - \eta)^n \cdot C_0 + (1 - \eta) \cdot \left[ \frac{1 - (1 - \eta)^n}{\eta} \right] \cdot F
\end{align*}
$$

$W_n + C_n = C_0 + \sum_{i=1}^{n} F_i$  All the revenues plus the initial credits have to be allocated between $L_n$ and $C_n$;

Substituting the previous relation, we obtain that the average of $F$ is equal to $F = \frac{\sum_{i=1}^{n} F_i}{n}$

and if $y = (1 - \eta)$

$$
\begin{align*}
C_n &= y^n \cdot C_0 + y \cdot \left[ \frac{1 - y^n}{1 - y} \right] \cdot \frac{\sum_{i=1}^{n} F_i}{n} = \\
y^{n+1} \cdot (C_0 + \frac{\sum_{i=1}^{n} F_i}{n}) - y^n \cdot C_0 + y^n \cdot (C_0 + \frac{\sum_{i=1}^{n} F_i}{n}) + C_n
\end{align*}
$$
Averages

Using the same technique we can define the average for:

- $\gamma, A$;
- $\omega, G$;
- $(nF - m.r), p/m, d$;
- $V, (Y' - Y)$;
XXX is a small mutual company which core business is based on microcredit and others mutual functions. XXX is a stockholder of YYY and utilizing the latter to finance both others Mutual Aid Societies and retail customers. Thus we can define its business similar to a normal consumer bank, in fact it receives funds from deposits and it invests in microcredit operations, acquiring shares from YYY.
XXX Model

\[
L_n = L_{(n-1)} + \bar{\eta}_n \cdot (C_{1(n-1)} + F_n) + \gamma_n \cdot K_{(n-1)} + \Omega \cdot Kf_{(n-1)} - \bar{\omega}_n \cdot (D_{(n-1)} + G_n) + \Delta f_n
\]

\[
C_n = (1 - \bar{\eta}_n) \cdot (C_{(n-1)} + F_n)
\]

\[
K_n = (1 - \gamma_n) \cdot K_{(n-1)} - A_n
\]

\[
Kf_n = (1 - \Omega_n) \cdot Kf_{n-1}
\]

\[
D_n = (1 - \bar{\omega}_n) \cdot (D_{(n-1)} + G_n)
\]

\[
Df_n = Df_{(n-1)} + n.F_n
\]

\[
\Delta f_n = n.F_n + m.r_n + p/m_n - d_n + \alpha_n
\]
From the Financial Statements available on-line we have:

\[
\begin{align*}
\text{Matrix} & & t_1 & t_2 & t_3 & t_4 & t_5 \\
\eta & 0.35 & 0.59 & 0.52 & 0.46 & 0.61 \\
\gamma & 0 & -0.62 & -22.7 & -0.032 & -0.56 \\
\Omega & 0.0068 & 0.21 & 0.71 & 0.048 & -0.001 \\
\omega & -0.75 & -0.70 & -0.75 & -0.83 & -0.81 \\
\end{align*}
\]

\[
\begin{align*}
\text{Vectors} & & t_1 & t_2 & t_3 & t_4 & t_5 & S_{03} \\
F & 99.977 & 183.079 & 211.585 & 206.335 & 221.449 & L_0 & 365.697 \\
G & 122.281 & 190.617 & 211.187 & 205.062 & 207.256 & C_0 & 66.506 \\
\Delta f & 244.658 & 275.819 & 105.524 & -25.988 & 152.509 & K_0 & 8.266 \\
n.F & 226.169 & 254.969 & 87.424 & -75.113 & 113.909 & D_0 & 18.152 \\
\end{align*}
\]
### XXX Averages

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Vectors</th>
<th>$S_{03}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>$F$</td>
<td>184.485</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$G$</td>
<td>187.280</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$\Delta f$</td>
<td>150.785</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$A$</td>
<td>$-5.148$</td>
</tr>
<tr>
<td></td>
<td>$n.F$</td>
<td>121.471</td>
</tr>
<tr>
<td></td>
<td>$L_0$</td>
<td>365.697</td>
</tr>
<tr>
<td></td>
<td>$C_0$</td>
<td>66.506</td>
</tr>
<tr>
<td></td>
<td>$K_0$</td>
<td>8.266</td>
</tr>
<tr>
<td></td>
<td>$Kf_0$</td>
<td>1.501.080</td>
</tr>
<tr>
<td></td>
<td>$D_0$</td>
<td>18.152</td>
</tr>
<tr>
<td></td>
<td>$Df_0$</td>
<td>1.265.434</td>
</tr>
</tbody>
</table>
Model in Average

Using this close formula

\[
S_n = M^n \cdot S_0 + \sum_{l=1}^{n} M^l \cdot CE + \sum_{l=0}^{n-1} M^l \cdot F
\]

we obtain:

\[
\begin{align*}
S_{08} & = 341.072 \\
L_{08} & = 341.072 \\
C_{08} & = 158.391 \\
K_{08} & = 295.875 \\
K_{f08} & = 1.952.615 \\
D_{08} & = 48.135 \\
D_{f08} & = 1.872.792 \\
\end{align*}
\]
Applying the typical financial ratio to XXX we can say that its financial situation does not seem very steady. In fact we have:

<table>
<thead>
<tr>
<th></th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
<th>$t_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Quick ratio</em></td>
<td>0.242</td>
<td>0.72</td>
<td>0.205</td>
<td>0.242</td>
<td>0.260</td>
</tr>
<tr>
<td><em>Net worth to FR</em></td>
<td>0.435</td>
<td>0.565</td>
<td>0.311</td>
<td>0.348</td>
<td>0.35</td>
</tr>
<tr>
<td><em>Current ratio</em></td>
<td>0.242</td>
<td>0.72</td>
<td>0.205</td>
<td>0.242</td>
<td>0.260</td>
</tr>
</tbody>
</table>

Ratios are not that good. The problems are the amount of financial debts and their temporal outlook, they are considered payable “within the 12 months”.
We can start by noticing that in our constant model there is a huge gap between the value of $\bar{n.F}$ given by our Chisini’s averages, replicating the Balance sheet in the period 2003 - 2008, which is $\bar{n.F} = 121.471$, and the same Chisini’s averages value of $\bar{n.F}$ that will bring our firm to a zero cash flow value in the year 2008 which is $\bar{n.F} = 49.541$ Euro. This could suggest that maybe the situation of our firm XXX is not that bad after all. But let’s get a little bit deeper into this subject through the next example.
We assume this is the budgeting model that has been planned by the management for the period 2010 - 2014.

<table>
<thead>
<tr>
<th></th>
<th>$S_{10}$</th>
<th>$S_{11}$</th>
<th>$S_{12}$</th>
<th>$S_{13}$</th>
<th>$S_{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$</td>
<td>385.653</td>
<td>422.013</td>
<td>449.535</td>
<td>467.753</td>
<td>526.563</td>
</tr>
<tr>
<td>$C$</td>
<td>159.484</td>
<td>159.993</td>
<td>160.230</td>
<td>160.340</td>
<td>74.580</td>
</tr>
<tr>
<td>$K$</td>
<td>290.727</td>
<td>285.579</td>
<td>280.431</td>
<td>275.283</td>
<td>270.135</td>
</tr>
<tr>
<td>$Kf$</td>
<td>2.058.065</td>
<td>2.169.210</td>
<td>2.286.358</td>
<td>2.409.832</td>
<td>2.539.974</td>
</tr>
<tr>
<td>$Df$</td>
<td>1.994.263</td>
<td>2.115.735</td>
<td>2.237.206</td>
<td>2.358.678</td>
<td>2.480.150</td>
</tr>
</tbody>
</table>

In this case ratios would keep on giving us a bad image on the hypothetical life of firm XXX for the period 2010 - 2014.
Constant Model

Our CM (constant model) describes the year 2014 with the following matrix and vectors of parameters in Chisini averages:

\[
\begin{bmatrix}
1 & 0.534 & 0 & -0.024 & -0.795 & 0 \\
0 & 0.466 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1.054 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.204 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

where we remind that the Chisini average for the case of \( n.F \) is simply the arithmetic mean \( \overline{n.F} = \frac{1}{h} \sum_{i=1}^{h} n.F_i \) (= 121.471 in our case).
Now we look for the value of $n.F$ that makes our cash flow equal to zero in the year 2014, this is 16.159, 1;

We would like to underline that there is a very important link between C.M. and V.M. (Variable Model). Given the parameters coming from our Budgeting Model year by year for V.M. (except for $n.F_h$) the C.M. replicates perfectly V.M. and vice versa for every set of parameters $n.F_1 .... n.F_h$ such that $\sum_{i=1}^{h} n.F_i = n.F$;

This is the reason why we can use only $n.F$ in order to analyse the behaviour of our Cash flow. Always remembering that we are in the case where the other parameters are all given according to the supposed Budgeting Model.
A probabilistic Approach

- Now we suppose that $n.F_h$ is a Random Variable for every step $h$;
- We have seen that our C.M. uses the arithmetic mean of $n.F_h$, thus the Random Variable we are concerned with will be $\frac{\sum_{i=1}^{n} F_i}{n} = \overline{n.F}$;
- Because of the Central Limit Theorems family we will assume that our R.V. will have a Gaussian distribution;
A probabilistic Approach

- The density function that we assume will be set on a significant number of values coming from the historical Financial Statements of our firm and others from similar firms;
- The average and the St. deviation of the statistical set we used are $\mu = 180.218, 5 \sigma = 55.955, 32$;
- Given the aforementioned distribution the probability that $n.F$ is less than 16.159, 1 (so that the Cash flow is less than zero in the year 2014) will be 0.1684%;
- According to our analysis the probability of a company distress appears to be really low, a conclusion that would not definitly be drawn looking at ratios.
We hope that the results shown above, could became the starting point of a new routes of research on this environment and we would like to think they can be also a starting point for a new view in the Management literature.